A Tutor™ for CASAS GOALS 2 Student Edition

Aztec Paxen Publishing

Math

Level D

Sampler

Steck-Vaughn Adult Education

- Aligned to CASAS Math GOALS 2 and CCRS
- Pretest to identify strengths and weaknesses
- Step-by-step instruction and targeted practice to boost CASAS Math GOALS 2 scores
- Full-length practice test to build learners' confidence

Aztec Paxen Publishing

Math

A Tutor™ for CASAS GOALS 2
Student Edition

Level D

Trademarks: The *A Tutor* Series is a trademark of Aztec Software, LLC. *A Tutor™ for CASAS* is not endorsed or approved by CASAS.

Image credits: iStock.com/wildpixel (cover), Максим Лебедик - stock.adobe.com (p. 198)

ISBN: 979-8-89141-062-6

Copyright by Aztec Software, LLC. All rights reserved. No part of this work may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying or recording, or by any information storage or retrieval system, without the prior written permission of the copyright owner, unless such copying is expressly permitted by federal copyright law. Requests for permission to make copies of any part of the work should be addressed to permissions@aztecsoftware.com or Aztec Software, LLC, 461 Headquarters Plaza North Tower, 2nd Floor, Morristown, NJ, 07960.

Printed in the U.S.A.

334 PX2040 34 33 32 31 30 29 28 27 26 25

A B C D E F

Contents

About CASAS GOALS v

Using *A Tutor™ for CASAS Math GOALS 2 Student Edition* vi

Unit 1 Number Sense and Operations: Rational Numbers

Lesson 1 Understand Negative Values 6
Lesson 2 Opposites 9
Lesson 3 Compare and Order Integers 12
Lesson 4 Converting Rational Numbers 15
Lesson 5 Compare and Order Rational Numbers 18
Lesson 6 Absolute Value 21
Lesson 7 Absolute Value and Order 24
Lesson 8 The Coordinate Plane 27
Lesson 9 Add Rational Numbers 30
Lesson 10 Subtract Rational Numbers 33
Lesson 11 Multiply and Divide Rational Numbers 36
Lesson 12 Word Problems with Rational Numbers 39
Lesson 13 Absolute Value on the Coordinate Plane 42
Lesson 14 Properties of Integer Exponents 45
Unit 1 Review 48

Unit 2 Number Sense and Operations: Ratios, Proportions, and Percentages

Lesson 15 Equivalent Ratios 51
Lesson 16 Solve Problems Involving Unit Rates 54
Lesson 17 Percent of a Quantity 57
Lesson 18 Find the Whole From a Percent 60
Lesson 19 Proportional Relationships: Apply Percents and Ratios 63
Lesson 20 Proportional Relationships 66
Lesson 21 Constant of Proportionality 69
Lesson 22 Direct Variation 72
Lesson 23 Graphing Proportional Relationships and Unit Rate 75
Unit 2 Review 78

Unit 3 Algebraic Thinking

Lesson 24 Adding and Subtracting Linear Expressions 81
Lesson 25 Expanding and Factoring Linear Expressions 84
Lesson 26 Equivalent Linear Expressions 87
Lesson 27 Multi-Step Problem Solving with Rational Numbers 90
Lesson 28 Solve Multi-Step Inequalities 93

Lesson 29 Multi-Step Equations 96
Lesson 30 Slope of Lines 99
Lesson 31 Constructing Linear Graphs from Tables and Equations 102
Lesson 32 Constructing Linear Graphs from Two Points 105
Lesson 33 Describing the Rate of Change and the Initial Value of a Linear Function 108
Lesson 34 Qualitative Graphs 111
Lesson 35 Solve Systems of Linear Equations by Graphing 114
Lesson 36 Solve Systems of Linear Equations Algebraically 117
Lesson 37 Square and Cubic Roots 120
Unit 3 Review 123

Unit 4 **Geometry and Measurement**

Lesson 38 Converting Measurements 126
Lesson 39 Interpret Scale Drawings 129
Lesson 40 Solve Problems Involving Scale 132
Lesson 41 Complementary and Supplementary Angles 135
Lesson 42 Parallel Lines and Transversals 138
Lesson 43 Angles of Triangles 141
Lesson 44 Transformations and Congruence 144
Lesson 45 Transformations and Similarity 147
Lesson 46 The Pythagorean Theorem 150
Lesson 47 Area and Circumference of Circles 153
Lesson 48 Area of Rectangles, Squares, Parallelograms, and Triangles 156
Lesson 49 Determine Volume 159
Lesson 50 Determine Surface Area 162
Unit 4 Review 165

Unit 5 **Data Analysis and Statistics**

Lesson 51 The Shape of Data 168
Lesson 52 Populations and Samples 171
Lesson 53 Comparing Data Distributions 174
Lesson 54 Compare Populations 177
Lesson 55 Scatter Plots: Clusters and Outliers 180
Lesson 56 Scatter Plots and Linear Associations 183
Lesson 57 Two-Way Tables 186
Unit 5 Review 189

Unit 6 Probability

Lesson 58 Understand Probability 192
Lesson 59 Use Probability 195
Lesson 60 Uniform Probability 198
Lesson 61 Probability 201
Lesson 62 Compound Events 204
Unit 6 Review 207

Answers and Explanations 217
Pretest Analysis Chart 253
Unit Review Analysis Charts 255
Practice Test Analysis Chart 267

About CASAS Math GOALS 2

CASAS stands for the Comprehensive Adult Student Assessment Systems. *GOALS* stands for Greater Opportunities for Adult Learning Success. The CASAS Math GOALS 2 tests are available in paper-and-pencil formats and online formats. CASAS Math GOALS 2 tests align with the CASAS Content Standards and the Adult Education College and Career Readiness Standards (AE-CCR). The CASAS Math GOALS 2 tests measure progress on the basic skills that adults need to succeed on the job and in life.

There are five levels for the CASAS Math GOALS 2:

CASAS Math GOALS 2 Levels A, B, C, D, E

Test	Number of Items (approximate)	Maximum Allowable Testing Time
Math GOALS 2 Level A	33	50 minutes
Math GOALS 2 Level B	36	65 minutes
Math GOALS 2 Level C	36	75 minutes
Math GOALS 2 Level D	36	75 minutes
Math GOALS 2 Level E	36	90 minutes

These tests measure basic adult math skills and the ability to make calculations and solve word problems that relate to real-world situations and traditional academic contexts. The CASAS Math GOALS 2 test includes multiple-choice items with four answer options. Basic calculators are allowed. Formulas are provided.

The math tests include items from the following domains.

CASAS Math GOALS 2 Content Areas	Covered Domains				
	A	B	C	D	E
Number Sense and Operations	✦	✦	✦	✦	✦
Algebraic Thinking	✦	✦	✦	✦	✦
Geometry	✦	✦	✦	✦	✦
Measurement	✦	✦	✦		
Data Analysis	✦	✦	✦		✦
Statistics		✦	✦	✦	✦
Probability			✦	✦	✦

Using *A Tutor™ for CASAS Math GOALS 2 Student Edition*

Step-by-Step Instruction

Each lesson starts with step-by-step instruction on a math concept or skill. The instruction contains examples and guided practice items with feedback. This instruction is followed by independent practice items. All items are aligned to a College and Career Readiness Standard and a CASAS Competency. Math items address real-life scenarios, workplace topics, and academic contexts. Work all of the items in the lesson's practice and then check your work in the Answers and Explanations section in the back of the book.

Pretest

Begin by taking the pretest on pages 1–5. Use the Pretest Analysis Chart on page 253 to identify your areas of strength and the areas you need to review. The chart will refer you to specific lessons in this book for further study.

Unit Reviews

Unit reviews give you a chance to check your understanding and determine if you have mastered the skills taught in each unit. Use the corresponding Unit Review Analysis Charts on pages 255–265 to identify strengths and weaknesses. These charts will help you determine if you need to review any of the lessons before continuing to the next unit.

Strategies and Hints

Pay careful attention to the Strategies and Hints throughout this book. Strategies are test-taking tips that help you do better on the test. Hints give you extra information about a skill.

Practice Test

At the end of the book, there is a full-length practice test. This test is similar to the CASAS Math GOALS 2 test. It has the same number and type of questions. This assessment will give you an idea of what the real test is like. Use the Practice Test Analysis Chart on pages 267–268 to identify areas you need to review.

Answers and Explanations

The answer key includes explanations for the correct answers to all practice items. After completing a lesson or unit review, check your answers on pages 217–252. For each item answered incorrectly, read the provided explanation and try reworking the item to obtain the correct answer. Please remember, there may be more than one way to solve a math problem. The explanations show one method of solving a problem. You may have used a different method than the one provided to correctly answer the question.

Additional Practice with *A Tutor™ for CASAS Math GOALS 2 Workbooks*

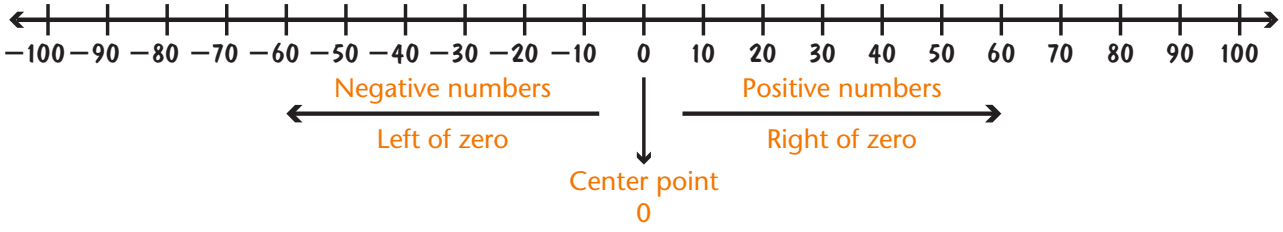
A Tutor™ for CASAS Math GOALS 2 Student Editions are designed to be used hand-in-hand with *A Tutor™ for CASAS Math GOALS 2 Workbooks*, which include practice sets for each lesson in the student editions. Each practice set provides additional items similar to those found on the CASAS Math GOALS 2 test.

- Use the practice sets after completing the corresponding student edition lesson if you feel you need additional practice.
- Use the practice sets after completing the unit reviews or Practice Test in the student edition for areas that need review and improvement.

Lesson 1 Understand Negative Values

6.NS.5

The set of whole positive numbers (1, 2, 3,…), whole negative numbers (–1, –2, –3,…), and zero are called **integers**. It can be helpful to think of a number line. On a horizontal number line, the numbers to the right of zero are positive numbers. They are greater than zero. The numbers to the left of zero are negative numbers. They are less than zero. The number zero is the center point and is neither positive nor negative.



Positive and negative numbers are used in the real world every day. They can be used to represent distance or temperature in relation to a reference point or to represent positive and negative changes. The table shows a list of common key words that will indicate if the number being spoken about is a positive number or negative number.

Represent Positive Numbers	Represent Negative Numbers
above	below
gain	loss
deposit	withdraw
credit	debit
earn	spend
elevate	descend
up	down
rise	fall

Using Positive and Negative Numbers for Measurement

Negative numbers show values that are less than 0. Remember that a positive number that is far from zero has a greater value than a positive number that is closer to zero. When used for measurements, the same is true for negative numbers. A negative number that is far from zero has a greater value than a negative number that is closer to zero. For most measurements, negative values are not used. Something cannot be –4 centimeters long or hold –12 cups of water.

Temperature measurements are different. A temperature scale always needs a reference point. This is where the 0 (zero) is defined. In both the Fahrenheit and Celsius systems, positive numbers are used to describe temperatures above zero. Negative numbers are used to describe temperatures below zero.

Distance can also be positive or negative. Like temperature, distance is measured based on a reference point. Using the center point as the zero on a number line, you can describe distances to the right as positive numbers and distances to the left as negative numbers. Likewise, sea level is considered the zero on a distance scale, so distances above sea level are positive numbers, and distances below sea level are negative numbers.

Example **Jeremy is 9 meters above the lobby of a hotel. Kaley is in the parking garage that is 12 meters below the hotel's lobby. Describe Jeremy's and Kaley's distance from the lobby using integers.**

The lobby represents 0, so positive integers are used to represent the distance above the lobby and negative integers are used to represent distances below the lobby.

Jeremy's distance from the lobby can be expressed as 9 meters. Kaley's distance from the lobby of the museum can be expressed as −12 meters.

Example **Giselle goes hiking. The table shows her elevation above sea level at various times throughout the day. What integer describes Giselle's change in elevation between 6 a.m. and 8 a.m.? What integer represents Giselle's change in elevation between 8 a.m. and 10 a.m.?**

Time	Elevation
6 a.m.	1,540 ft
8 a.m.	1,760 ft
10 a.m.	1,690 ft
12 p.m.	1,480 ft

1) Determine the magnitude of the change by calculating the difference between the two elevations:

$1{,}760 - 1{,}540 = 220$
$1{,}760 - 1{,}690 = 70$

2) Determine whether the change represents an increase or decrease in elevation for each interval.
From 6 am. to 8 a.m., Giselle's elevation is increasing, so the change is represented with a positive number: 220 ft.
From 8 a.m. to 10 a.m., Giselle's elevation is decreasing, so the change is represented with a negative number: –70 ft.

Guided Practice

Joan is scuba diving 25 feet below sea level. What integer represents the location of Joan?

A. 25 feet
B. 15 feet
C. −25 feet
D. −20 feet

Option C is correct. Joan is below the starting point of sea level. The word "below" lets us know that it is a negative number. Because she is 25 feet below sea level, it is represented as −25 feet.

Practice

Read each question. Select the correct answer.

1 Jane is standing on a ladder 2 feet above the ground. Which integer represents Jane's location in feet?

A. –2 **B.** 0
C. 1 **D.** 2

2 Renee is 10 meters below sea level taking photographs of a coral reef. Which integer represents Renee's location in meters?

A. 15 **B.** 10
C. −10 **D.** −15

3 Which table shows the correct integers representing each situation?

A.

Situation	Integer
5 students dropped out of a class	−5
a withdraw of $100 from a bank account	100
a $5 increase in hourly pay	5
a deposit of $100 into a bank account	−100

B.

Situation	Integer
5 students dropped out of a class	5
a withdraw of $100 from a bank account	−100
a $5 increase in hourly pay	5
a deposit of $100 into a bank account	100

C.

Situation	Integer
5 students dropped out of a class	−5
a withdraw of $100 from a bank account	−100
a $5 increase in hourly pay	−5
a deposit of $100 into a bank account	100

D.

Situation	Integer
5 students dropped out of a class	−5
a withdraw of $100 from a bank account	−100
a $5 increase in hourly pay	5
a deposit of $100 into a bank account	100

4 The football team was on the 10-yard line; they gained 15 yards. What integer represents the gained yards?

A. 25 **B.** −15
C. 10 **D.** 15

5 Sabrina goes for a run in the afternoon when the temperature is 68 degrees. In the evening, she goes for a walk when the temperature is 54 degrees. Which integer represents the change in temperature?

A. 24 degrees **B.** 14 degrees
C. −14 degrees **D.** −24 degrees

6 Caleb weighed 145 pounds in November and 152 pounds the following January. Which integer represents the change in Caleb's weight?

A. 7 pounds **B.** 2 pounds
C. −2 pounds **D.** −7 pounds

7 A stock closes with a $10.25 value on Monday. On Tuesday, the net change in value is −$0.25. What is the closing price of the stock on Tuesday?

A. $10.75 **B.** $10.50
C. $10.25 **D.** $10.00

8 Phil runs a marathon in 4 hours and 25 minutes one year. The next year, he runs it in 4 hours and 12 minutes. Which integer represents the change in Phil's time?

A. −25 minutes **B.** −13 minutes
C. 13 minutes **D.** 25 minutes

Use the information and table to answer questions 9 and 10.

The table shows the temperature of Sadie's oven one afternoon.

Time	Temperature
4:00	79 degrees
4:10	140 degrees
4:20	375 degrees
4:30	375 degrees
4:45	375 degrees
5:00	300 degrees
5:10	160 degrees
5:20	90 degrees
5:30	82 degrees

9 Which integer represents the change in temperature between 5:00 and 5:10?

A. 61 degrees **B.** 0 degrees
C. −75 degrees **D.** −140 degrees

10 Over which interval would the temperature change be represented with a positive integer?

A. 4:10 to 4:20 **B.** 4:20 to 4:30
C. 4:45 to 5:00 **D.** 5:20 to 5:30

Lesson 15 Equivalent Ratios

6.RP.3, 6.RP.3.a

Equivalent ratios are two ratios that express the same relationship between the numbers in each ratio. You can create ratios that are equivalent to a given ratio by multiplying or dividing both numbers that form the given ratio by the same number.

Example **A food nutrition label states that 1 serving is 4 ounces. Complete the table of equivalent ratios.**

Servings	1				
Ounces	4				

If 1 serving contains 4 ounces, then

- 2 servings must contain twice as many ounces, or $2 \cdot 4 = 8$ ounces,
- 3 servings must contain three times as many ounces, or $3 \cdot 4 = 12$ ounces,
- 4 servings must contain four times as many ounces, or $4 \cdot 4 = 16$ ounces, and
- 5 servings must contain five times as many ounces, or $5 \cdot 4 = 20$ ounces.

	Original ratio	$2 \cdot 1$ ↓	$3 \cdot 1$ ↓	$4 \cdot 1$ ↓	$5 \cdot 1$ ↓
Servings	1	2	3	4	5
Ounces	4	8	12	16	20
		↑ $2 \cdot 4$	↑ $3 \cdot 4$	↑ $4 \cdot 4$	↑ $5 \cdot 4$

Because the numbers 1 and 4 in the initial ratio are both multiplied by the same number to complete each column in the table, 2:8, 3:12, 4:16, and 5:20 are all ratios equivalent to 1:4.

Example **Use equivalent ratios to find the unknown value.**

$$\frac{5}{11} = \frac{y}{88}$$

1) Multiplying both terms of a ratio by the same number produces an equivalent ratio. Find the number that relates the denominators 11 and 88.

$$\frac{5}{11 \cdot 8} = \frac{y}{88}$$

2) To produce an equivalent ratio, the 5 in the first ratio must also be multiplied by 8.

$$\frac{5 \cdot 8}{11 \cdot 8} = \frac{40}{88}$$

$$\frac{5}{11} = \frac{40}{88}$$

The unknown value is $y = 40$.

Example **Raul has a family recipe that serves a large group of people. The recipe calls for 10 pounds of ground beef for 25 servings. He wants to make the same recipe but for less people. What an equivalent ratio that will create the fewest servings?**

1) Write the ratio of pounds of ground beef per number of servings as a fraction.

$$\frac{\text{pounds of ground beef}}{\text{number of servings}} = \frac{10}{25}$$

2) Determine the Greatest Common Factor of the two values in the ratio.

10 = 1, 2, 5, 10

25 = 1, 5, 25

5 is the GCF of the values in the ratio.

3) Divide both values by 5 to create an equivalent ratio.

$$\frac{10 \div 5}{25 \div 5} = \frac{2}{5}$$

Raul can use the ratio 2 pounds of ground beef for every 5 servings to create the same recipe for fewer people. Note: If he needs 10 servings, he could use 4 pounds of ground beef. If he needs 15 servings, he could use 6 pounds of ground beef.

Example **An assemble-it-yourself kit requires 9 screws to complete. The kit includes the 9 screws, along with 2 extras. Which of the two ratios 4:16 and 6:27 is equivalent to the ratio of extra screws to required screws?**

1) Compare the ratios 2:9 and 4:16.

2 times the first term of 2:9 is the first term of 4:16.

2 times the second term of 2:9 is not the second term of 4:16.

$$\frac{2}{9} \times \frac{2}{2} \neq \frac{4}{16}$$

You cannot obtain the ratio 4:16 by multiplying the terms of 2:9 by the same number. The ratio 4:16 is not equivalent to 2:9.

2) Compare the ratios 2:9 and 6:27.

3 times the first term of 2:9 is the first term of 6:27.

3 times the second term of 2:9 is the second term of 6:27.

$$\frac{2}{9} \times \frac{3}{3} = \frac{6}{27}$$

You can obtain the ratio 6:27 by multiplying the terms of 2:9 by the same number. The ratio 6:27 is equivalent to 2:9.

Guided Practice

A deli makes fresh juice blends. One blend has a fixed ratio of orange juice to mango juice. The table shows the amounts of juice in the blend sizes served to the last four customers:

Customer	Mango Juice (ounces)	Orange Juice (ounces)
1	3	6
2	6	14
3	2	4
4	4	8

One of the portions was blended incorrectly. Which customer was *not* served the same ratio as the others?

A. Customer 1 **B.** Customer 2

C. Customer 3 **D.** Customer 4

Option B is correct. The ratio of mango juice to orange juice served to Customers 1, 3, and 4 is 1:2. The ratio served to Customer 3 is not equivalent to 1:2. If the ratio had been equivalent, then 6 ounces of mango juice would need to have 12 ounces of orange juice added.

Customer 1: $\frac{3 \div 3}{6 \div 3} = \frac{1}{2}$. Customer 2: $\frac{6 \div 2}{14 \div 2} = \frac{3}{7}$. Customer 3: $\frac{2 \div 2}{4 \div 2} = \frac{1}{2}$. Customer 4: $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$.

Practice

Read each question. Select the correct answer.

1 A ball falls to the ground at a constant rate. The table shows the relationship between the time in minutes and the distance in kilometers the ball falls. What is the value of y?

Time (minutes)	Area (kilometers)
1	5
3	y
5	25

A. $y = 15$ **B.** $y = 12$

C. $y = 24$ **D.** $y = 10$

2 A camel walks at a constant rate. The table shows the relationship between the time in hours and the distance in miles. What is the value of x?

Time (hours)	Distance (miles)
1	12
2	24
x	180
20	240

A. $x = 6$ **B.** $x = 15$

C. $x = 7$ **D.** $x = 3$

3 In a bin, there is a mix of 6 pink pens for every 8 purple pens. Which ratio is equivalent to the ratio of purple pens to pink pens?

A. $\frac{12}{8}$ **B.** $\frac{12}{6}$

C. $\frac{20}{12}$ **D.** $\frac{24}{18}$

4 Which ratios are equivalent?

A. $\frac{15}{3}$ and $\frac{30}{6}$

B. $\frac{13}{8}$ and $\frac{26}{21}$

C. $\frac{8}{9}$ and $\frac{11}{12}$

D. $\frac{4}{7}$ and $\frac{7}{21}$

5 A shelter has an adoption rate of 4 dogs for every 5 cats. Which ratio is equivalent to the ratio of cats to dogs?

A. 5:7

B. 15:12

C. 21:15

D. 16:14

Lesson 24 Adding and Subtracting Linear Expressions

7.EE.1

A linear expression contains only constants and variables raised to the power of 1. They have the general form of $mx + b$, where m and b can be any number. The table shows examples of expressions of that are linear and expressions that are not linear.

Linear Expressions	Not Linear Expressions
$5x + 20$	$\frac{1}{x}$
36	$6x^2 + \frac{4}{5}$
$x - 2.8$	$1 + 5^x$
$-83x$	$9.3 - x^3$
$\frac{3}{4}x - \left(\frac{8}{2}x + 2\right)$	

When adding or subtracting linear expressions, the constants should be combined, and the variables should be combined. However, a constant can never be combined with a variable.

$3 - 9$	Combine the constants to make -6.
$\frac{1}{2}x + \frac{1}{4}x$	Combine the variables to make $\frac{3}{4}x$.
$8 + 9x$	These terms cannot be combined.

Adding a Linear Expression

If a linear expression includes parentheses, the first step is to remove the parentheses. Then, identify any like terms and combine them.

Subtracting a Linear Expression

When subtracting a linear expression, keep track of negative signs when rearranging and combining terms. Be sure to distribute the negative sign across any terms in parentheses.

Example **Simplify $9x + 18 + 14x$.**

1) Identify like terms. The expression has only one constant term, 18. It has two variable terms, $9x$ and $14x$.

2) Use the Commutative Property to gather like terms. $9x + 14x + 18$

3) Use the Distributive Property to combine the variable terms. $(9 + 14)x + 18$
$23x + 18$

The simplified form of the expression is $23x + 18$.

Example **Simplify $(6.3x + 12) + (3x + 17)$.**

1) Remove the parentheses. $6.3x + 12 + 3x + 17$

2) Identify like terms. The expression has two constant terms, 12 and 17. It has two variable terms, $6.3x$ and $3x$.

3) Use the Commutative Property to gather like terms. $6.3x + 3x + 12 + 17$

4) Add the constant terms. $6.3x + 3x + 29$

5) Use the Distributive Property to combine the variable terms. $(6.3 + 3)x + 29$

The simplified form of the expression is $9.3x + 29$. $9.3x + 29$

Example **Simplify $\left(\frac{3}{7}x + 45\right) - \left(\frac{1}{7}x + 31\right)$.**

1) Remove the parentheses. Distribute the negative sign across each term in parentheses. $\frac{3}{7}x + 45 - \frac{1}{7}x - 31$

2) Identify like terms. The expression has two constant terms, 45 and 31. It has two variable terms, $\frac{3}{7}x$ and $\frac{1}{7}x$.

3) Use the Commutative Property to gather like terms, distributing the negative sign to both terms in parentheses. $\frac{3}{7}x - \frac{1}{7}x + 45 - 31$

4) Subtract the constant term 31. $\frac{3}{7}x - \frac{1}{7}x + 14$

5) Use the Distributive Property to combine the variable terms $\frac{3}{7}x - \frac{1}{7}x + 14$

The simplified form of the expression is $\frac{2}{7}x + 14$. $\frac{2}{7}x + 14$

Vertical Addition and Subtraction of Linear Expressions

Another way to add and subtract linear expressions is to rewrite the expressions in columns. Be sure the constant terms are aligned and the variable terms are aligned.

Example **Simplify $(16 - 12x) - (4x + 31)$.**

1) Write the expressions in a column with like terms aligned. Notice that the positions of 16 and $-12x$ must be switched to align the variable terms and align the constant terms.

$$\begin{array}{r} -12x + 16 \\ \underline{-(4x + 31)} \end{array}$$

2) Distribute the negative sign to both terms in parentheses. Subtract like terms.

$$\begin{array}{r} -12x + 16 \\ \underline{\ -4x - 31} \\ -16x - 15 \end{array}$$

The simplified form of the expression is $-16x - 15$.

Guided Practice

Simplify $(6x + 8.3) + (-9 - 5x)$.

A. $x - 0.7$

B. $-x - 0.7$

C. $3x + 3.3$

D. $-3x + 3.3$

Option A is correct. Option A is correct. Remove the parentheses.

$6x + 8.3 - 9 - 5x$

Use the Commutative Property to gather like terms.

$6x - 5x + 8.3 - 9$

Add the constant terms.

$6x - 5x - 0.7$

Use the Distributive Property to combine the variable terms. Simplify.

$(6 - 5)x - 0.7 = x - 0.7$

Practice

Read each question. Select the correct answer.

1 What is the simplified form of the expression?

$(4x + 5) + (7x + 9)$

A. $-3x - 4$

B. $9x + 16$

C. $11x + 14$

D. $13x + 12$

2 What is the simplified form of the expression?

$(7x + 2) - (15x + 9)$

A. $-8x - 7$

B. $22x + 11$

C. $-2x - 13$

D. $9x - 24$

3 What is the simplified form of the expression?

$(12.8x + 3) + (9.1x + 8)$

A. $3.7x - 5$

B. $21.9x + 11$

C. $15.8x + 17.1$

D. $20.8x + 12.1$

4 What is the simplified form of the expression?

$\left(\frac{3}{8} + 6x\right) + \left(23x + \frac{7}{8}\right)$

A. $17x + 1\frac{1}{4}$

B. $17x + \frac{3}{4}$

C. $29x + \frac{3}{4}$

D. $29x + 1\frac{1}{4}$

5 What is the simplified form of the expression?

$(5.8x - 8) - (19x + 4.2)$

A. $24.8x - 3.8$

B. $-13.2x - 12.2$

C. $10x - 27$

D. $12.2x + 24.8$

Lesson 38 Converting Measurements

6.RP.3.d

You can use ratios and reasoning about equivalent ratios to convert units of measurement. A **conversion factor** is a ratio that gives the relationship between two different units. A conversion factor is ALWAYS equal to 1.

For example, in the ratio $\frac{24 \text{ hours}}{1 \text{ day}}$, if you replace 1 day with its equivalent, 24 hours, it is clear that $\frac{24 \text{ hours}}{24 \text{ hours}}$ equals 1.

Some other conversion factors include $\frac{32 \text{ ounces}}{1 \text{ quart}}$, $\frac{2.54 \text{ centimeters}}{1 \text{ inch}}$, and $\frac{5{,}280 \text{ feet}}{1 \text{ mile}}$.

When using conversion factors, it is crucial to include all units. When converting between measurement systems, such as from the metric system to the customary system, the conversion factors will be provided to you or can be found online.

Sometimes, you will need to use more than one conversion factor to solve a problem. Once you have written all the conversion factors, multiply them to convert the units of measurement.

Example **A golf ball travels 330 meters. What is this distance in kilometers?**

1) Begin with a ratio that puts the information you have in the numerator and the number 1 in the denominator. $\frac{330 \text{ meters}}{1}$

When converting from meters to kilometers, you go from a smaller measure of distance to a larger measure of distance (1 meter < 1 kilometer). This means that the answer (in kilometers) will be *less than* 330 (in meters).

2) Multiply the original ratio by the conversion factor. Write the conversion factor so that the unit for your final answer (kilometers) is in the numerator. $\frac{330 \text{ meters}}{1} \times \frac{1 \text{ kilometer}}{1{,}000 \text{ meters}}$

3) Notice that the unit *meters* appears in a numerator and a denominator. Because $\frac{\text{meter}}{\text{meter}} = 1$ and multiplying by 1 leaves a number unchanged, you can eliminate (or cancel) this unit. $\frac{330 \text{ } \cancel{\text{meters}}}{1} \times \frac{1 \text{ kilometer}}{1{,}000 \text{ } \cancel{\text{meters}}}$

$\frac{330 \times 1 \text{ kilometer}}{1{,}000}$

4) Simplify.

$\frac{330 \times 1 \text{ kilometer}}{1{,}000} = \frac{330 \text{ kilometer}}{1{,}000} = 0.33 \text{ kilometer}$

So, the golf ball travels 0.33 kilometer.

Example **How many minutes are in 5 days?**

1) Write a ratio with a denominator of 1 that contains the known information in the numerator.

$$\frac{5 \text{ days}}{1}$$

You are going from a larger period of time to a smaller period of time (1 day > 1 hour > 1 minute), so your answer (in minutes) will be *greater than* 5 (in days).

2) Multiply the original ratio by each conversion factor needed. The new unit for your final answer (minutes) will be in the numerator.

$$\frac{5 \text{ days}}{1} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{60 \text{ minutes}}{1 \text{ hour}}$$

3) Because two conversion factors are needed, you will eliminate two units.

$$\frac{5 \text{ } \cancel{\text{days}}}{1} \times \frac{24 \text{ } \cancel{\text{hours}}}{1 \text{ } \cancel{\text{day}}} \times \frac{60 \text{ minutes}}{1 \text{ } \cancel{\text{hour}}}$$

$$\frac{5 \times 24 \times 60 \text{ minutes}}{1}$$

4) Simplify.

$$\frac{5 \times 24 \times 60 \text{ minutes}}{1} = 7{,}200 \text{ minutes}$$

So, 5 days is the same as 7,200 minutes.

Example **A group of 6 friends are on a morning hike. They are each carrying 24 ounces of water. How many liters of water are they carrying in all?**

1) Determine how many total ounces of water the friends took on the hike.

$$6 \text{ friends} \times 24 \text{ ounces} = 144 \text{ ounces}$$

2) Because liters is a metric measure and ounces is a customary measure, identify the conversion factor for these units that is needed: 1 liter is about 33.8 ounces.

$$\frac{1 \text{ liter}}{33.8 \text{ ounces}}$$

3) Multiply the original ratio by each conversion factor needed. The new unit for your final answer (minutes) will be in the numerator.

$$\frac{144 \text{ ounces}}{1} \times \frac{1 \text{ liter}}{33.8 \text{ ounces}}$$

4) Eliminate ounces in the numerator of the first ratio and the denominator of the conversion factor.

$$\frac{144 \text{ } \cancel{\text{ounces}}}{1} \times \frac{1 \text{ liter}}{33.8 \text{ } \cancel{\text{ounces}}}$$

$$\frac{144 \times 1 \text{ liter}}{33.8}$$

5) Simplify.

$$\frac{144 \times 1 \text{ liter}}{33.8} = \frac{144 \text{ liter}}{33.8} \approx 4.26 \text{ liters}$$

Since the conversion factor is not an exact amount, the answer is an approximate amount. The group of friends is carrying about 4.26 liters of water in all.

Guided Practice

A soup recipe calls for 0.75 kilogram of mushrooms. At the grocery store, Logan finds mushrooms in half-pound packages. How many packages of mushrooms should Logan buy to be sure to have enough? (One pound is equivalent to about 0.454 kilogram.)

A. 5 packages

B. 4 packages

C. 6 packages

D. 3 packages

Option B is correct. Begin with a ratio that puts the information you have in the numerator and the number 1 in the denominator: $\frac{0.75 \text{ kg}}{1}$. Multiply the original ratio by the conversion factor to convert kilograms to pounds. Then use a conversion factor to convert to the unit for your final answer (packages) is in the numerator: $\frac{0.75 \text{ kg}}{1} \times \frac{1 \text{ pound}}{0.454 \text{ kg}} \times \frac{1 \text{ package}}{0.5 \text{ pound}}$. Eliminate any units that appear in both a numerator and denominator.

$\frac{0.75 \text{ kg}}{1} \times \frac{1 \text{ pound}}{0.454 \text{ kg}} \times \frac{1 \text{ package}}{0.5 \text{ pound}}$

$\frac{0.75 \times 1 \times 1 \text{ package}}{1 \times 0.454 \times 0.5} = \frac{0.75 \text{ package}}{0.227} \approx 3.30$ packages

Since the conversion from kilograms to pounds is an estimate, the answer is an approximation. Because the number of packages must be a whole number, Logan needs to purchase 4 packages of mushrooms to make the recipe.

Practice

Read each question. Select the correct answer.

1 Joseph uses 33 inches of electrical tape to wrap an exposed wire. How many feet of electrical tape does he use? (1 foot = 12 inches)

A. 2.3 ft

B. 3.25 ft

C. 2.75 ft

D. 3.35 ft

2 Casey is painting one wall in her garage. The dimensions of the wall are 15 feet long by 5 feet tall. What are the dimensions of the wall in yards? (1 yard = 3 feet)

A. $4\frac{1}{4}$ yd by 3 yd

B. $3\frac{1}{3}$ yd by $2\frac{1}{2}$ yd

C. $3\frac{2}{3}$ yd by $2\frac{1}{3}$ yd

D. 5 yd by $1\frac{2}{3}$ yd

3 Which measurement is equivalent to 4 yards? (1 yard = 3 feet, 1 foot = 12 inches)

A. 7 ft

B. 9 ft

C. 96 in.

D. 144 in.

4 Amanda is traveling to Germany and wants to convert her money from US dollars to euros. She has $1,500. About how much money does she have in euros? ($1 ≈ 0.93 €)

A. 1,395 €

B. 1,424 €

C. 1,519 €

D. 1,612 €

5 Tyler is baking matzah bread for dinner. The recipe calls for 3 cups of flour. Tyler only has a tablespoon to use to scoop the flour. What is the amount of flour that Tyler needs in tablespoons? (1 cup = 16 tablespoons)

A. 16 Tbsp

B. 24 Tbsp

C. 32 Tbsp

D. 48 Tbsp

Lesson 51 The Shape of Data

6.SP.5.d

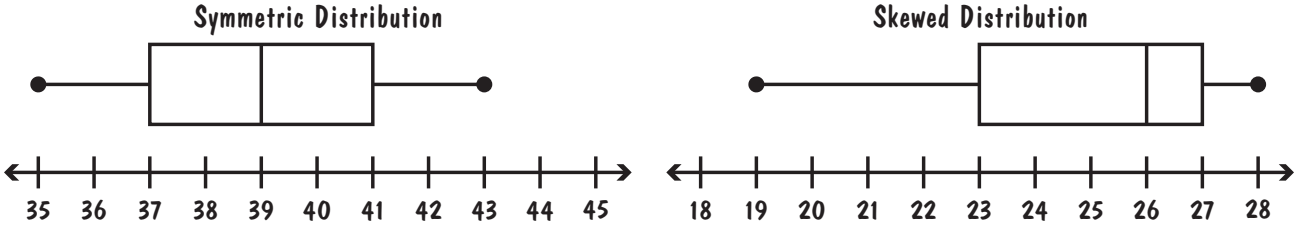
Two properties of data sets that are helpful in understanding the data are the measure of center and the measure of variability. The **measure of center** is the central value, which includes the mean, the median, and the mode. The **mean** is the average value. The **median** is the middle value. The **mode** is the value that appears most often in the set.

The **measure of variability** is a description of how spread out the values of the data set are. The **interquartile range (IQR)** is a measure based on the middle half of the data set. The **mean absolute deviation (MAD)** is a measure based on the average distance between each value and the mean.

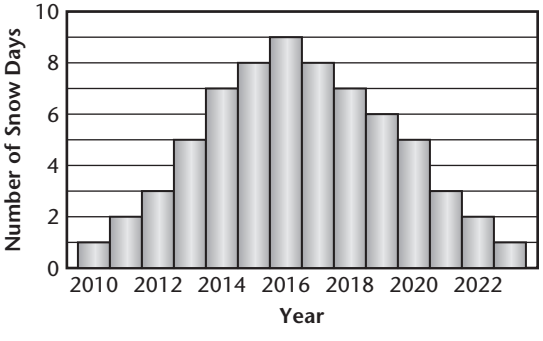
The shape of a data distribution usually indicates which measure of center and which measure of variability are more meaningful representations of the data. However, when choosing which to use to describe a data set, you should first consider the context in which the data were gathered. In some cases, the context can indicate that some of the data should be excluded from the measures.

Symmetrical and Skewed Distributions

The distribution of a data set is **symmetrical** if the values are evenly distributed around a central value. The mean is generally the best measure of center, and the MAD is the best measure of variability to describe a symmetrical distribution. The distribution of a data set is **skewed** if it trails off much more on one side than the other. The median is generally the best measure of center, and the IQR is the best measure of variability to describe a skewed distribution.



Example **The graph shows the number of snow days a city had per year over a 13-year period. Which measure of variability *best* represents the number of snow days?**



1) Analyze the graph. The data appear to be symmetrically distributed around a central value.

2) Identify possible measures of variability. Consider the interquartile range (IQR) and the mean absolute deviation (MAD) as possible measures of variability.

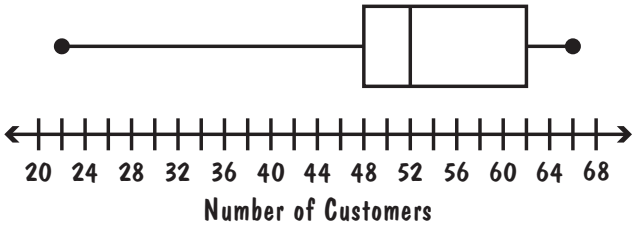
3) Determine which measure of variability best represents the data according to the shape of the distribution. The IQR is useful for skewed distributions because it considers only the middle half

of the data. The MAD is useful for symmetrical distributions because it considers all the data. Because this distribution is fairly symmetrical, the MAD best represents the variability of the number of snow days.

Consideration of the Context

In some cases, the context clearly shows that you should not rely on one or more data values. If the unreliable data are far away from all other data, you can infer that the data are errors. You should not include those values when deciding which measure of center or measure of variability to use. However, not all outliers are errors. Only ignore data if the information clearly does not represent the purpose for which the data were collected.

Example **The manager of a small deli recorded the number of lunch customers each day for a week to determine a typical number of customers to expect. A strong thunderstorm on one of the days likely affected the number of customers that day. The graph represents the data. Which measure of center *best* represents the number of lunch customers each day at the deli?**



1) Analyze the graph. The data range from 22 to 66. The box represents the middle half of the values. The vertical bar in the box shows the median.

2) Identify possible measures of variability. Consider the mean and median as possible measures of center to describe the data.

3) Consider the context in which the data were collected. The graph represents only seven data values. A thunderstorm could significantly decrease the number of customers, so it is likely that just that one value has skewed the data left. The box shows that most of the data are far above the minimum value. Because the vertical bar is far to the left of the box, it is likely that just one data value draws the median far left of the center.

4) Determine which measure of center best represents the data according to the shape of the distribution. Because the manager wants to know how many customers to expect each day, the best option is to ignore the data for the stormy day and calculate the mean for the remaining days.

Guided Practice

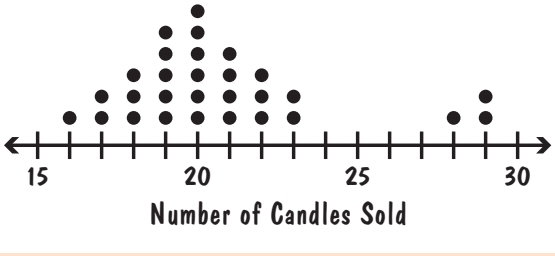
The graph shows the number of homemade candles that Elena sold each day at a craft fair over the summer. Which measure of variability *best* represents the data?

A. the MAD of all the data

B. the MAD with the data greater than 23 removed

C. the IQR of all the data

D. the IQR with the data greater than 23 removed

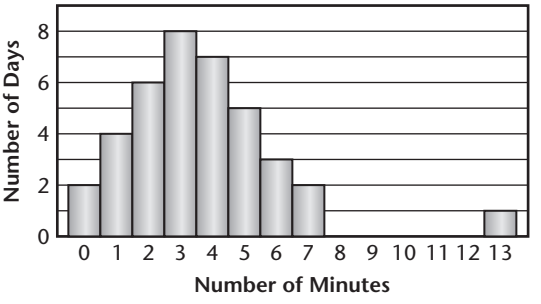


Option C is correct. The context does not indicate any justification for excluding any of the data from consideration. Because the data are skewed, the IQR is the better measure of variability.

Practice

Read each question. Select the correct answer.

1 Zane created a graph to show the number of minutes it took him to get coffee from the break room. On one of the days, he stopped to talk to a coworker. What is the *best* measure of variability for the data?



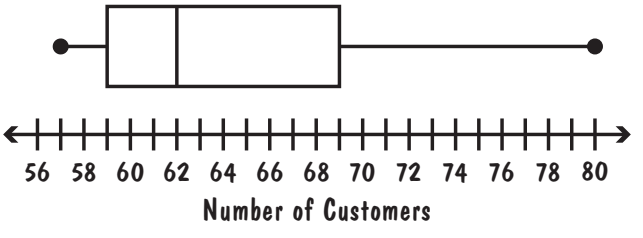
A. the MAD of all the data

B. the MAD with the data greater than 7 minutes removed

C. the IQR of all the data

D. the IQR with the data greater than 7 minutes removed

2 The graph shows the number of customers served lunch at a food truck each weekday. What is the *best* measure of center for the data?



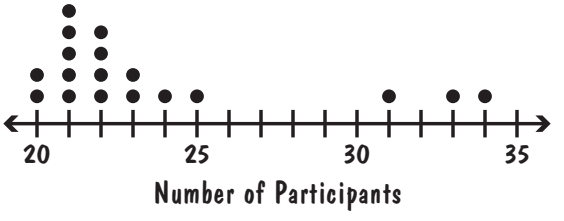
A. mean

B. median

C. IQR

D. MAD

3 The graph shows the number of participants in a volleyball tournament each year. What is the *best* measure of variability for the data?



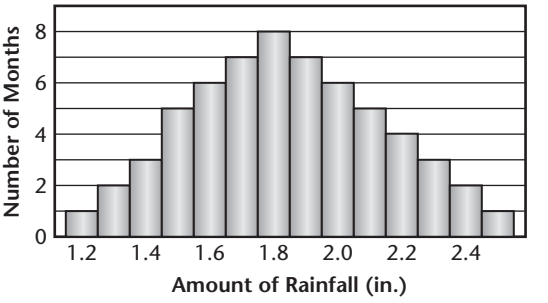
A. mean

B. median

C. IQR

D. MAD

4 The graph shows the monthly rainfall in a city over a 5-year period. What is the *best* measure of variability for the data?



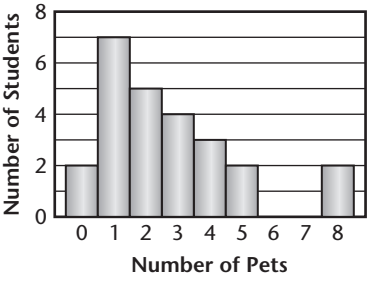
A. mean

B. median

C. IQR

D. MAD

5 The graph shows the number of pets that people in an office have. What is the *best* measure of center for the data?



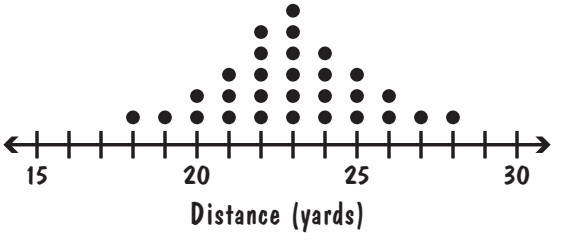
A. mean

B. median

C. IQR

D. MAD

6 The graph shows the distances parents were able to throw a football in a competition. What is the *best* measure of center for the data?



A. mean

B. median

C. IQR

D. MAD

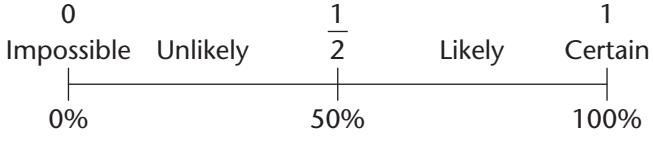
Lesson 58 Understand Probability

7.SP.5

The **probability** of an event is the chance of it occurring. An event can be described and labeled as impossible, unlikely, neither likely nor unlikely, likely, or certain. Probabilities range between 0 and 1, or between 0% and 100%. An event that has a probability of 0 on the continuum is impossible. A probability of 1 means the chance of the event happening is certain.

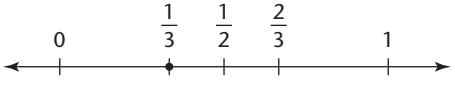
Probabilities that fall between these two numbers are part of a continuum. They will either be likely, unlikely, or neither likely nor unlikely depending upon where they fall on the scale. The closer the probability is to 1 (or 100%), the more likely the event is to occur. The closer the probability is to 0 (or 0%), the less likely the event is to occur. A probability of 0.5 (or 50%) is neither likely nor unlikely to occur. Probabilities can be expressed as decimals, fractions, or percentages.

Probability Continuum



Example **May heard on the news that the probability of rain tomorrow is $\frac{1}{3}$. Is rain tomorrow impossible, unlikely, neither likely nor unlikely, likely, or certain?**

1) Locate the number $\frac{1}{3}$ on the number line.



2) Because the fraction is not 0, the probability of rain is not impossible. It is also not certain because the probability is not 1. The fraction $\frac{1}{3}$ is closer to 0 than 1, which falls in the unlikely category on the continuum. Rain tomorrow is unlikely to occur.

Example **Gio is the only person competing in his karate division. Is his chance of winning his division impossible, unlikely, neither likely nor unlikely, likely, or certain?**

1) Because Gio is the only person competing, the probability of him winning his division is 1.

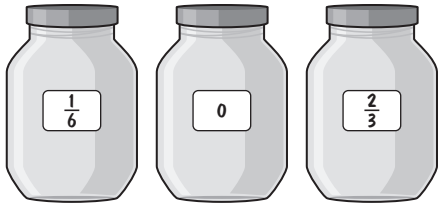
2) Locate 1 on the probability continuum. A probability of 1 falls in the certain category on the continuum. Gio is certain to win his division.

Example **Melony is sick and is unable to compete in the swimming competition. Is her chance of winning the swimming competition impossible, unlikely, neither likely nor unlikely, likely, or certain?**

1) Because Melony is sick and cannot compete, the probability of her winning the competition is 0.

2) Locate 0 on the probability continuum. A probability of 0 falls in the impossible category on the continuum. It is impossible for Melony to win the swimming competition.

Example **Monica's children like to play with marbles. Her three children all prefer the blue marbles. Monica put some blue marbles and some white marbles into each of three jars. She told her children to choose a jar, close their eyes, and reach in and take a marble. She labeled each jar with the probability of pulling out a blue marble. Which jar should her children choose for the *best* chance of getting a blue marble?**



1) Order the probabilities of getting a blue marble from least to greatest: 0, $\frac{1}{6}$, $\frac{2}{3}$.

2) Because $\frac{2}{3}$ is the greatest probability, Monica's children should choose Jar 3 for the best chance of getting a blue marble.

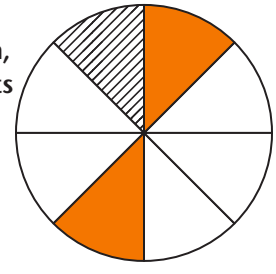
Determining Simple Probabilities

The **probability** of an event is the ratio of the number of desired outcomes to the total number of possible outcomes.

Consider a six-sided number cube. The cube has 6 faces, so the total number of possible outcomes when rolling the number cube is 6. Each face shows a different number of dots, or a different number: 1, 2, 3, 4, 5, or 6.

So, the probability of rolling a 1 is $\frac{1}{6}$. Similarly, the probability of rolling a 2 is $\frac{1}{6}$. All outcomes are equally likely. So, the probabilities of rolling a 3, a 4, a 5, or a 6 are each also $\frac{1}{6}$.

Example **During game night, Chauhan and his friends play a game that involves spinning a spinner. If the spinner lands on the striped section, the player loses a turn. If it lands on an orange section, the player gets to spin again. If it lands on a white section, the player has to answer a question. Is it impossible, unlikely, neither likely nor unlikely, likely, or certain that Chauhan will have to answer a question after his first spin?**



1) There are 8 possible outcomes on this spinner. Determine the probability of landing on each section: white, orange, and striped.

- There are 5 white sections on the spinner, so the probability of the spinner landing on white is $\frac{5}{8}$.
- There are 2 orange sections on the spinner, so the probability of the spinner landing on orange is $\frac{2}{8}$, or $\frac{1}{4}$.
- There is 1 striped section on the spinner, so the probability of the spinner landing on striped is $\frac{1}{8}$.

2) White sections represent the event that a player will have to answer a question. So, locate $\frac{5}{8}$ on the probability continuum. Because $\frac{5}{8}$ is between $\frac{1}{2}$ and 1 on the continuum, it is likely that Chauhan will have to answer a question after his first spin.

Guided Practice

Raelynn saw on the news that her favorite baseball team has a $\frac{2}{5}$ probability of winning their game tomorrow. Is a win for the team impossible, unlikely, likely, or certain?

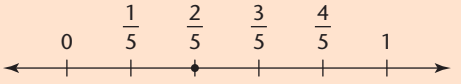
A. impossible

B. unlikely

C. likely

D. certain

Option B is correct. Locate the number $\frac{2}{5}$ on the number line.



The fraction $\frac{2}{5}$ is closer to 0 than to 1. This falls into the unlikely category on the continuum. So, a win for the team is unlikely.

Practice

Read each question. Select the correct answer.

1 Kalen read in the newspaper that there is a 0.50 probability of it snowing on Friday. Which statement *best* describes the chance of snow on Friday?

A. It is likely that it will snow on Friday.

B. It is certain that it will snow on Friday.

C. It is neither likely nor unlikely that it will snow on Friday.

D. It is unlikely that it will snow on Friday.

2 There are three buffet lines at a hospital cafeteria. Each visitor is given a random piece of fruit from a fruit bowl.

- The bowl in line 1 contains 6 apples and 4 oranges.
- The bowl in line 2 contains 3 apples and 9 oranges.
- The bowl in line 3 contains 11 apples and 4 oranges.

In which buffet line should a visitor go through to have the *best* chance of getting an apple?

A. line 3

B. line 2

C. line 1

D. impossible

3 Saul learned in science class that the Earth rotates on its axis each day. Is the probability of the Earth rotating on its axis tomorrow impossible, unlikely, likely, or certain?

A. impossible

B. unlikely

C. likely

D. certain

4 Mandy's favorite football player has a probability of 0.84 of being MVP of the game. Is the player's chance of being MVP impossible, unlikely, likely, or certain?

A. impossible

B. unlikely

C. likely

D. certain

5 Cooper has played 10 tennis matches and has not won any of them. Based on his record so far, which statement *best* describes Cooper's chance of winning his next match?

A. It is impossible for Cooper to win his next match.

B. Cooper's record shows that he has not won any matches so far, it is very unlikely that he will win the next one. However, it is not impossible.

C. Cooper will certainly win his next match because his luck has to change.

D. Cooper is neither likely nor unlikely to win his next match.