A Tutor™ for CASAS GOALS 2 Student Edition

Aztec Paxen Publishing

Math

Level E

Sampler

Steck-Vaughn Adult Education

- Aligned to CASAS Math GOALS 2 and CCRS
- Pretest to identify strengths and weaknesses
- Step-by-step instruction and targeted practice to boost CASAS Math GOALS 2 scores
- Full-length practice test to build learners' confidence

Aztec Paxen Publishing

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Level E

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About CASAS Math GOALS 2

CASAS stands for the Comprehensive Adult Student Assessment Systems. *GOALS* stands for Greater Opportunities for Adult Learning Success. The CASAS Math GOALS 2 tests are available in paper-and-pencil formats and online formats. CASAS Math GOALS 2 tests align with the CASAS Content Standards and the Adult Education College and Career Readiness Standards (AE-CCR). The CASAS Math GOALS 2 tests measure progress on the basic skills that adults need to succeed on the job and in life.

There are five levels for the CASAS Math GOALS 2:

CASAS Math GOALS 2 Levels A, B, C, D, E

Test	Number of Items (approximate)	Maximum Allowable Testing Time
Math GOALS 2 Level A	33	50 minutes
Math GOALS 2 Level B	36	65 minutes
Math GOALS 2 Level C	36	75 minutes
Math GOALS 2 Level D	36	75 minutes
Math GOALS 2 Level E	36	90 minutes

These tests measure basic adult math skills and the ability to make calculations and solve word problems that relate to real-world situations and traditional academic contexts. The CASAS Math GOALS 2 test includes multiple-choice items with four answer options. Basic calculators are allowed. Formulas are provided.

The math tests include items from the following domains.

CASAS Math GOALS 2 Content Areas

	Covered Domains				
	A	B	C	D	E
Number Sense and Operations	✦	✦	✦	✦	✦
Algebraic Thinking	✦	✦	✦	✦	✦
Geometry	✦	✦	✦	✦	✦
Measurement	✦	✦	✦		
Data Analysis	✦	✦	✦		✦
Statistics		✦	✦	✦	✦
Probability			✦	✦	✦

Using *A Tutor™ for CASAS Math GOALS 2 Student Edition*

Step-by-Step Instruction

Each lesson starts with step-by-step instruction on a math concept or skill. The instruction contains examples and guided practice items with feedback. This instruction is followed by independent practice items. All items are aligned to a College and Career Readiness Standard and a CASAS Competency. Math items address real-life scenarios, workplace topics, and academic contexts. Work all of the items in the lesson's practice and then check your work in the Answers and Explanations section in the back of the book.

Pretest

Begin by taking the pretest on pages 1–5. Use the Pretest Analysis Chart on page 233 to identify your areas of strength and the areas you need to review. The chart will refer you to specific lessons in this book for further study.

Unit Reviews

Unit reviews give you a chance to check your understanding and determine if you have mastered the skills taught in each unit. Use the corresponding Unit Review Analysis Charts on pages 235–246 to identify strengths and weaknesses. These charts will help you determine if you need to review any of the lessons before continuing to the next unit.

Strategies and Hints

Pay careful attention to the Strategies and Hints throughout this book. Strategies are test-taking tips that help you do better on the test. Hints give you extra information about a skill.

Practice Test

At the end of the book, there is a full-length practice test. This test is similar to the CASAS Math GOALS 2 test. It has the same number and type of questions. This assessment will give you an idea of what the real test is like. Use the Practice Test Analysis Chart on page 247 to identify areas you need to review.

Answers and Explanations

The answer key includes explanations for the correct answers to all practice items. After completing a lesson or unit review, check your answers on pages 195–231. For each item answered incorrectly, read the provided explanation and try reworking the item to obtain the correct answer. Please remember, there may be more than one way to solve a math problem. The explanations show one method of solving a problem. You may have used a different method than the one provided to correctly answer the question.

Additional Practice with *A Tutor™ for CASAS Math GOALS 2 Workbooks*

A Tutor™ for CASAS Math GOALS 2 Student Editions are designed to be used hand-in-hand with *A Tutor™ for CASAS Math GOALS 2 Workbooks*, which include practice sets for each lesson in the student editions. Each practice set provides additional items similar to those found on the CASAS Math GOALS 2 test.

- Use the practice sets after completing the corresponding student edition lesson if you feel you need additional practice.
- Use the practice sets after completing the unit reviews or Practice Test in the student edition for areas that need review and improvement.

Test-Taking Tips

- Read the directions very carefully.
- Read each question carefully. Make sure you know what you have to do.
- Read all of the answer options carefully, even if you think you know the answer.
- Answer all of the questions. If you cannot find the right answer, rule out the answers that you know are wrong. Then try to figure out the right answer. If you still do not know, make your best guess.
- Do not change an answer unless you are sure your first answer is wrong. Usually, your first idea is the correct answer.
- If you get nervous, stop and take a few breaths. Then start working again.

Lesson 1 Proportional Relationships Between Units of Linear Measure

6.RP.3.d

Measurements are used to find length, weight, or capacity of an object. Two of the most common measurement systems are the customary measurement system and the metric measurement system. To convert between each of the systems, you can either multiply by a conversion factor or write and solve a proportion. Use the following conversion factors to find equivalent measurements.

Customary to Metric
1 inch = 2.54 centimeters
1 foot ≈ 0.31 meter
1 yard ≈ 0.91 meter
1 mile ≈ 1.61 kilometers

Metric to Customary
1 centimeter ≈ 0.39 inch
1 meter ≈ 3.28 feet
1 meter ≈ 1.09 yards
1 kilometer ≈ 0.62 mile

Example **A euro banknote is 3.2 inches wide and 6.3 inches long. Approximately how many centimeters wide and long is the euro? Use 1 in. ≈ 2.54 cm**

1) Write the conversion factor as a ratio: $\frac{2.54 \text{ centimeters}}{1 \text{ inch}}$

2) Use the conversion factor to convert the given measurements.

width: $3.2 \text{ } \cancel{\text{inches}} \times \frac{2.54 \text{ centimeters}}{1 \text{ } \cancel{\text{inch}}} = \frac{3.2 \times 2.54 \text{ centimeters}}{1} \approx 8.13$ centimeters

length: $6.3 \text{ } \cancel{\text{inches}} \times \frac{2.54 \text{ centimeters}}{1 \text{ } \cancel{\text{inch}}} = \frac{6.3 \times 2.54 \text{ centimeters}}{1} \approx 16.00$ centimeters

The euro is approximately 8.13 centimeters wide and 16.00 centimeters long.

Example **A spool contains 85 meters of electrical wire. Use a proportion to approximate how many feet of wire the spool contains. Use 1 m ≈ 3.28 ft.**

1) Use a proportion with the conversion ratio on one side and a ratio that includes x as the unknown measurement on the other side.

$$\frac{1 \text{ meter}}{3.28 \text{ ft}} = \frac{85 \text{ meters}}{x \text{ feet}}$$

2) Solve the proportion. Cross multiply, and solve for the unknown.

$$\frac{1 \text{ meter}}{3.28 \text{ feet}} = \frac{85 \text{ meters}}{x \text{ feet}}$$

$$1x = 85 \times 3.28$$

$$x \approx 278.8$$

The spool contains approximately 278.8 feet of electrical wire.

Example **Katie Ledecky is famous for her dominance in the 1,500-meter freestyle swimming competition. How many miles is 1,500 meters? Use 1,000 m = 1 km and 1 km ≈ 0.62 mi.**

1) There is not a conversion fact that goes directly from meters to miles, so first convert meters to kilometers and then convert kilometers to miles.

2) Write the conversion factor for meters to kilometers as a ratio with meters in the denominator: $\frac{1 \text{ kilometer}}{1{,}000 \text{ meters}}$.

3) Use the conversion factor to convert meters to kilometers: $1{,}500 \text{ } \cancel{\text{meters}} \times \frac{1 \text{ kilometer}}{1{,}000 \text{ } \cancel{\text{meters}}} = 1.5$ kilometers.

4) Write the conversion factor for kilometers to miles as a ratio with kilometers in the denominator: $\frac{0.62 \text{ mile}}{1 \text{ kilometer}}$.

5) Use the conversion factor to convert kilometers to miles: $1.5 \text{ } \cancel{\text{kilometers}} \times \frac{0.62 \text{ mile}}{1 \text{ } \cancel{\text{kilometer}}} \approx 0.92$ mile.

The 1,500-meter race is 0.92 mile.

Notice that the conversion factor is a ratio with the unit you are converting to in the numerator and the unit you are converting from in the denominator.

Guided Practice

The walkway from the parking lot to the entrance of a park can be no longer than 42 yards. Approximately how many meters is the maximum allowed length of walkway? Use 1 yd ≈ 0.91 m.

A. 45.95 m

B. 45.78 m

C. 38.22 m

D. 12.81 m

Option C is correct. Create a proportion: $\frac{1 \text{ yard}}{0.91 \text{ meter}} = \frac{42 \text{ yards}}{x \text{ meters}}$.
Solve for the unknown: $x = 42 \times 0.91 \approx 38.22$.
The maximum length of walkway is 38.22 meters.

Strategy

When converting between measurement systems, you could solve a proportion with the unknown unit as a variable, or you can multiply the known measurement by a conversion factor represented as a ratio.

Practice

Read each question. Select the correct answer.

1 A craft store offers a box containing several rolls of washi tape. The total length of tape in the box is 520 feet. Approximately how many meters of tape does the box contain? Use 1 ft ≈ 0.31 m.

A. 161.20 meters

B. 475.28 meters

C. 569.93 meters

D. 1,704.90 meters

2 The local recreation department is sponsoring a baby crawling race. The length of the race is 5 meters. Approximately how many feet is the baby crawling race? Use 1 m ≈ 3.28 ft.

A. 11.57 feet

B. 13.76 feet

C. 16.40 feet

D. 21.40 feet

3 One World Trade Center located in New York, New York is 541 meters tall. Approximately how many yards tall is One World Trade Center? Use 1 m ≈ 1.09 yd.

A. 774.48 yards

B. 589.69 yards

C. 496.33 yards

D. 164.94 yards

4 The International Code Council issued updated regulations stating that bleachers and grandstands with stepped aisles must have aisles with a minimum width of 121.9 centimeters. What is the approximate minimum width in inches? Use 1 cm ≈ 0.39 in.

A. 37.16 inches

B. 47.54 inches

C. 312.56 inches

D. 399.83 inches

5 A safety sign located near a construction site warns pedestrians to keep a distance of at least 5 feet from the work area. Approximately how many meters is this distance? Use 1 ft ≈ 0.31 m.

A. 6.39 meters

B. 5.47 meters

C. 4.57 meters

D. 1.55 meters

6 Electrocardiogram machines are often used in medical offices and hospitals. One type of machine has a width of 38.1 centimeters. Approximately how many inches wide is the electrocardiogram machine? Use 1 cm ≈ 0.39 in.

A. 11.62 inches **B.** 14.86 inches

C. 97.69 inches **D.** 99.97 inches

7 The speed limit within city limits in Spain is 50 kilometers per hour. What is the speed limit in miles per hour? Use 1 kilometer ≈ 0.62 mile.

A. 22 miles per hour

B. 31 miles per hour

C. 46 miles per hour

D. 81 miles per hour

8 A European shoe of size 40 has a length of 24.6 centimeters. How long is a size 40 shoe in inches? Use 1 centimeter ≈ 0.39 inch.

A. 8.9 inches

B. 9.2 inches

C. 9.6 inches

D. 10.4 inches

9 An amusement park requires riders for the Boom Blaster to be 54 inches tall. How many centimeters tall must riders be? Use 1 inch ≈ 2.54 centimeters.

A. 21.3 centimeters

B. 108 centimeters

C. 122 centimeters

D. 137 centimeters

10 Paige and Joey climbed 5,000 feet of elevation to reach the top of the Half Dome at Yosemite National Park in California. How many meters of elevation did they gain? Use 1 foot ≈ 0.31 meter.

A. 1,280 meters

B. 1,490 meters

C. 1,550 meters

D. 1,613 meters

11 A candy company sells bubblegum tape in a dispenser that holds 3 feet of bubblegum. How many centimeters long is the bubblegum tape? Use 1 foot = 12 inches and 1 inch ≈ 2.54 centimeters.

A. 30.48 centimeters

B. 36 centimeters

C. 84.68 centimeters

D. 91.44 centimeters

12 Latisha's stride length is 2.3 feet. How many meters will she walk in 10 steps? Use 1 foot ≈ 0.31 meter.

A. 1.8 meters

B. 7.13 meters

C. 34.26 meters

D. 74.19 meters

Lesson 7 Add Polynomials

A.APR.1

An expression such as $3xy^5$ that has just one term is called a monomial. Expressions with two terms, such as $3xy^5 - \frac{2x}{7y}$, are called binomials. Expressions with three terms, such as $3xy^5 - \frac{2x}{7y} + x^{-3}$, are called trinomials. All of these expressions, no matter how many terms they have, are called **polynomials**.

Like terms in polynomials are terms whose variables and exponents are the same. $3a$, $\frac{1}{2}a$, $-9a$ are like terms because the variables are all a. The terms $7ab^2$, $\frac{1}{2}ab^2$, $-4ab^2$ are also like terms because the variables are all ab^2.

To add polynomials, add the like terms by adding the coefficients of those like terms. You cannot add unlike terms. The terms a and a^2 *cannot* be added. The terms a^2 and b^2 *cannot* be added. Only like terms can be added.

Example **Add $(3a^2 + 2a + 5) + (4a - 3)$.**

1) Rewrite the problem so that the like terms are lined up in columns.

$$\begin{array}{r} 3a^2 + 2a + 5 \\ +\quad\quad 4a - 3 \\ \hline \end{array}$$

2) Add the like terms by combining their coefficients.

$$\begin{array}{r} 3a^2 + 2a + 5 \\ +\quad\quad 4a - 3 \\ \hline 3a^2 + 6a + 2 \end{array}$$

Example **Add $(5a^2 - 4) + (2a^2 - 3a + 6)$.**

1) Rewrite the problem so that the like terms are lined up in columns.

$$\begin{array}{r} 5a^2 \quad\quad - 4 \\ 2a^2 - 3a + 6 \\ \hline \end{array}$$

2) Add the like terms by combining their coefficients.

$$\begin{array}{r} 5a^2 \quad\quad - 4 \\ +\ 2a^2 - 3a + 6 \\ \hline 7a^2 - 3a + 2 \end{array}$$

Example **Add $(b^2 - 2ab + 3a^2) + (2b^2 + 4ab - 5a^2)$.**

1) Rewrite by grouping like terms together.

$(b^2 + 2b^2) + [(-2ab) + 4ab] + [3a^2 + (-5a^2)]$

2) Add the like terms by combining their coefficients.

$3b^2 + 2ab - 2a^2$

Hint

Make sure you move the sign of the term when you regroup.

Example **A triangle has sides $2a + 3$, $3b + 4$, and $4a + 5b$. What is the perimeter of the triangle?**

1) The perimeter is the sum of all sides of a polygon. Add the sides.

$(2a + 3) + (3b + 4) + (4a + 5b) =$

2) Group like terms and add by combining coefficients.

$(2a + 4a) + (3b + 5b) + (3 + 4) = 6a + 8b + 7$

Example **A rectangle has sides $3a + 1$, $4b + 3$, $3a + 1$, and $4b + 3$. What is the perimeter of the rectangle?**

1) The perimeter is the sum of the sides: $(3a + 1) + (4b + 3) + (3a + 1) + (4b + 3)$.

2) Group like terms and add: $(3a + 3a) + (4b + 4b) + (1 + 3 + 1 + 3) = 6a + 8b + 8$.

Guided Practice

What is the sum of $(3a + 4ab - 2b) + (-5a + 3ab + 5b)$?

A. $-2a + 7ab + 3b$

B. $-2a + 7ab - 3b$

C. $2a + 7ab + 3b$

D. $2a + 7ab - 3b$

Option A is correct. Rewrite the expression by grouping, and add like terms.

$(3a + 4ab - 2b) + (-5a + 3ab + 5b)$

$= [3a + (-5a)] + (4ab + 3ab) + [-2b + 5b]$

$= -2a + 7ab + 3b$

Hint

You cannot simplify any further after combining like terms because unlike terms cannot be combined by adding or subtracting.

Practice

Read each question. Select the correct answer.

1 What is the sum of $(4a + 5b) + (-6a + 3b)$?

A. $2a + 8b$

B. $10a + 8b$

C. $-2a + 8b$

D. $-10a + 8b$

2 What is the sum of $(8a + 5b) + (4a - 6b - 3)$?

A. $12a - b - 3$

B. $12a - b + 3$

C. $12a + b - 3$

D. $12a + b + 3$

3 What is the sum of $(7a^2 - 5b^2 + 1) + (-4a^2 + 3b^2 - 5)$?

A. $3a^2 + 2b^2 - 6$

B. $3a^2 + 2b^2 - 4$

C. $3a^2 - 2b^2 - 6$

D. $3a^2 - 2b^2 - 4$

4 What is the sum of $(4a^2 + 3ab + 8b^2) + (-3a^2 + 5ab - 6b^2 + 2)$?

A. $a^2 + 8ab + 2b^2 + 2$

B. $a^2 + 8ab - 2b^2 + 2$

C. $7a^2 + 8ab + 2b^2 + 2$

D. $7a^2 + 8ab - 2b^2 + 2$

5 What is the sum of $(2a^2 - 3ab - 5b^2 + 6) + (4a^2 - 3ab + 5b^2 + 6)$?

A. $6a^2 + 12$

B. $6a^2 - 10b^2 + 12$

C. $6a^2 - 6ab + 12$

D. $6a^2 - 6ab - 10b^2 + 12$

6 What is the sum of $(4a^2 + 6b^2) + (-3a^2 - 2ab + 5b^2 - 8)$?

A. $a^2 - 2ab + b^2 - 8$

B. $a^2 - 2ab + 11b^2 - 8$

C. $7a^2 - 2ab + b^2 - 8$

D. $7a^2 - 2ab + 11b^2 - 8$

7 What is the sum of $(3a^2 - 2ab - 3) + (-4a^2 + 6b^2) + (8ab + 3b^2 - 5)$?

A. $-a^2 + 6ab + 9b^2 - 8$

B. $-a^2 + 6ab + 9b^2 - 2$

C. $a^2 + 6ab + 9b^2 - 8$

D. $a^2 + 6ab + 9b^2 - 2$

8 A triangle has sides $3a + 4b$, $6a - 2b$, and $-a + 3b$. What is the perimeter of the triangle?

A. $8a + 5b$ **B.** $9a + 5b$

C. $8a + 9b$ **D.** $9a + 9b$

9 A quadrilateral has sides $9a + ab$, $4b + 2ab$, $-2a + 3ab$, and $-b + 4ab$. What is the perimeter of the quadrilateral?

A. $11a + 5b + 10ab$

B. $11a + 3b + 10ab$

C. $7a + 5b + 10ab$

D. $7a + 3b + 10ab$

10 A triangle has sides with lengths $4x + y$, $3x^2 - 2y$, and $5x - 3y$. What is the perimeter of the triangle?

A. $3x^2 + 9x - 4y$

B. $12x - 4y$

C. $3x^2 + 9x + 6y$

D. $3x^2 - 4y$

11 A trapezoid has sides with lengths $m + n$, $m - 2n$, $2m + 3$, and $4n - 2$. What is the perimeter of the trapezoid?

A. $8m - n + 1$

B. $4m - 7n - 6$

C. $4m + 3n + 1$

D. $8m + 5n - 6$

12 Skye computes part of a problem that leads to the polynomial $6a^2 + 5a$. Her study partner, Latoya, computes the other part of the problem, which gives her the polynomial $10a^3 - 4a + 3$. What is the final solution to the problem, which is found by adding the two polynomials?

A. $16a^3 + a + 3$

B. $10a^3 + 6a^2 + 9a + 3$

C. $16a^2 + a + 3$

D. $10a^3 + 6a^2 + a + 3$

13 Brian has four investment funds, whose returns depend on several variables, such as a = federal prime interest rate, b = stock market index, and c = consumer spending. His four funds yield returns according to the polynomials $a + 3b + 4c$, $-4a + b + 2c$, $-a + 4b + c$, and $2a - b + c$. Which polynomial gives the total return for all of Brian's investment funds?

A. $2a - 7b - 8c$

B. $-2a + 7b + 8c$

C. $a + b + c$

D. $-3a + 6b + 7c$

14 Two measures of two adjacent angles are given by $a^2 + b + c^2$ and $a - b^2 - c$. Which polynomial is the sum of the two angle measures?

A. $2a^2 + 2b^2 + 2c^2$

B. $a^2 + a - b^2 + b + c^2 - c$

C. $2a + 2b + 2c$

D. $a^2 + a + b^2 + b + c^2 + c$

Lesson 20 Relationships Between Quantities

7.RP.2.a, 7.RP.2.b, 7.RP.2.c, 7.RP.2.d

A proportional relationship is a relationship where the ratio of one quantity to the other is constant. This relationship represented as $y = kx$ is called the **direct proportion equation**. The k is the **constant of proportionality** or **unit rate** for the proportional relationship, defined as $k = \frac{y}{x}$.

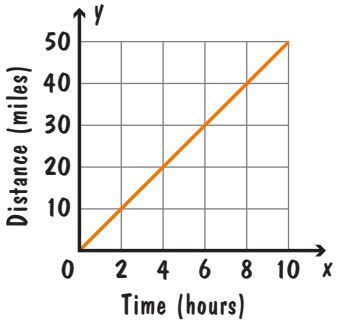
You can graph proportional relationships using the equation $y = kx$. When graphing proportional relationships, the unit rate is also known as the **slope**.

Example **The proportional relationship of distance traveled over time of 3 aquatic animals is shown. Which animal has the fastest speed in miles per hour (mph)?**

Bottlenose Dolphin

Time (hours)	Distance (miles)
2	40
3	60
4	80
5	100

King Penguin



Swordfish
$y = 60x$,
where x is the time (hours) and y is the distance (miles).

1) Determine the unit rate of travel for each animal by dividing the distance traveled (y) by the number of hours (x).

Bottlenose Dolphin: $\frac{40}{2} = 20$ mph

King Penguin: $\frac{50}{10} = 5$ mph

Swordfish: 60 mph

2) Compare and order the unit rates.

$5 < 20 < 60$

Because swordfish have the greatest unit rate, they have the fastest speed.

Example **Liam binge-watches his favorite show each night through an ad-free streaming service after work. The table shows the time it takes to watch a given number of episodes.**

Episodes	Time (minutes)
2	96
3	144
5	240

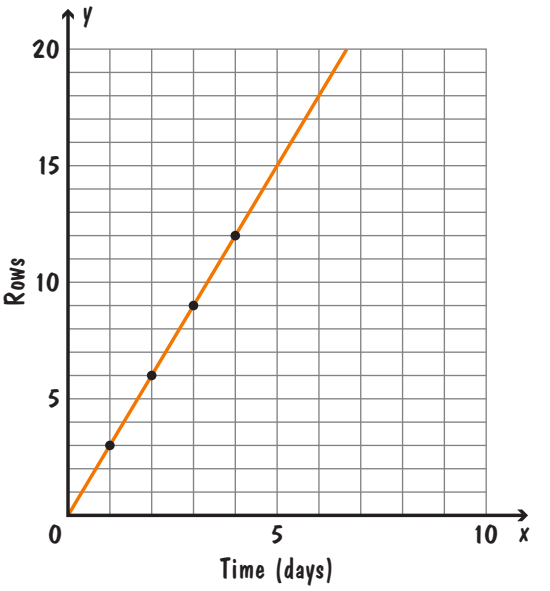
1) Write an equation to represent this proportional relationship. Directly proportional relationships can be expressed in an equation of the form $y = kx$. Determine the constant of proportionality by dividing any y-value by the corresponding x-value.

$k = \frac{y}{x} = \frac{96}{2} = 48$

2) Substitute the constant of proportionality into the equation $y = kx$: $y = 48x$.

Liam's TV time is represented by $y = 48x$, where x represents the number of episodes Liam watches, and y represents the number of minutes he watches.

Example **Hina is crocheting a baby blanket for her new niece. The graph shows the number of rows in the blanket as Hina works on it each day.**



1) Make a table of the coordinates of the points shown in the graph The *x*-value of each point represents the number of days Hina has been crocheting and the *y*-value represents the number of rows in the blanket. So, the point (1, 3) means that on the first day, Hina completes 3 rows. Enter the coordinates for the four points on the graph in the table.

Time (days)	Rows
1	3
2	6
3	9
4	12

2) Find Hina's daily rate of progress. The graph is a straight line that goes through the origin, so the relationship is proportional. Hina's daily progress is the unit rate and can be determined by dividing any *y*-value by the corresponding *x*-value: $\frac{12}{4} = \frac{9}{3} = \frac{6}{2} = \frac{3}{1} = 3$. No matter which ordered pair you use, you can describe Hina's daily rate of progress as 3 rows per day.

Guided Practice

The total cost of shirts is proportional to the number of shirts purchased. Store A charges \$116 for 4 shirts. The table shows the proportional relationship between the number of shirts and the cost at Store B. Which store has the lower price for shirts?

Number of Shirts	Total Cost (dollars)
3	69
5	115
9	207
16	368

A. Store A

B. Store B

C. Store A and Store B have the same price for shirts.

D. The relationship cannot be determined.

Option B is correct. Find the cost of 1 shirt for each store and compare.
Store A: \$116 ÷ 4 = \$29; Store B: \$69 ÷ 3 = \$23.
Store B has the lower price.

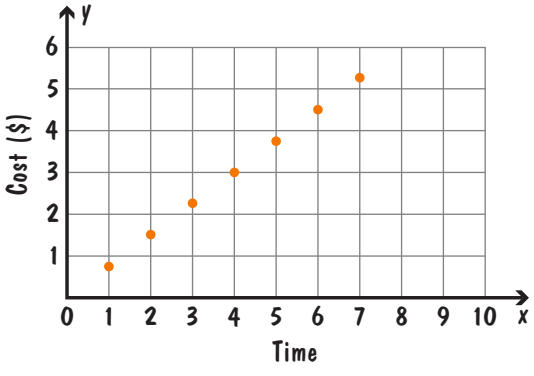
Strategy

To find the unit rate for Store B, choose one of the data points and divide the *y*-value by the *x*-value.

Practice

Read each question. Select the correct answer.

1 Which proportional relationships is represented on the graph?



A. Paying a fee for a 5-day overdue book costs $5.

B. Traveling 3 miles in a taxi costs $4.

C. Buying 6 bottles of water costs $27.

D. Making a 4-minute international call costs $3.

2 Which pair of points represent the same proportional relationship?

A. (2, 5) and (3, 8) B. (2, 5) and (8, 20)

C. (3, 8) and (8, 20) D. (3, 8) and (17, 34)

3 A therapist charges $75 for 25 minutes and $150 for 50 minutes. What is the unit rate?

A. $3.00 per minute B. $3.50 per minute

C. $4.00 per minute D. $4.50 per minute

4 Which equation represents the proportional relationship for the ordered pairs (9, 24) and (12, 32)?

A. $y = \frac{5}{3}x$ B. $y = \frac{7}{3}x$

C. $y = \frac{8}{3}x$ D. $y = 3x$

5 Which equation represents the proportional relationship for the ordered pairs (12, 9) and (36, 27)?

A. $y = \frac{3}{2}x$ B. $y = \frac{2}{3}x$

C. $y = \frac{4}{3}x$ D. $y = \frac{3}{4}x$

6 The total cost of firewood is proportional to the number of cords purchased. Company A charges $1,057 for 7 cords. The table shows the proportional relationship between the number of cords and the cost at Company B. Which company has the lower price per cord of firewood?

Firewood (number of cords)	Total Cost
0.5	$75
1.5	$225
2	$300
5	$750

A. Company A

B. Company B

C. Company A and Company B have the same price for firewood.

D. The relationship cannot be determined.

7 It costs $12.00 per hour to park in a city parking lot for 8 hours. It costs $10.50 to park in a parking garage for 6 hours. Which parking service offers the better rate?

A. the city parking lot because $1.50 per hour < $1.75 per hour

B. the city parking lot because $0.67 per hour > $0.57 per hour

C. the parking garage because $10.50 per hour < $12.00 per hour

D. the parking garage because $0.57 per hour < $0.67 per hour

8 Which table represents the proportional relationship $y = \frac{2}{3}x$?

A.

x	y
6	9
8	12
10	15

B.

x	y
9	6
12	8
15	10

C.

x	y
12	9
16	12
20	15

D.

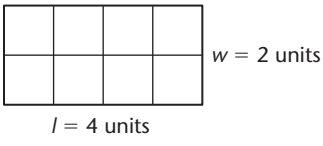
x	y
9	12
12	16
15	20

Lesson 33 Find Unknown Dimensions of Shapes

7.G.6

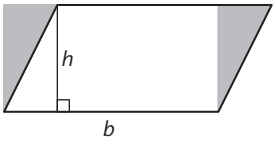
Area is the measure of the space inside a two-dimensional shape and is measured in square units. The area of a rectangle may be determined using the formula $A = lw$, where l is the length and w is the width.

$A = lw = 4 \times 2 = 8$ units^2

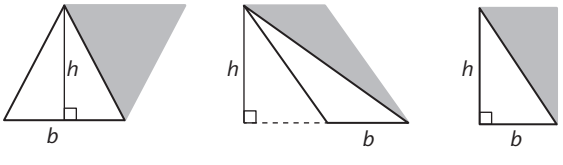


Parallelograms may have consecutive sides that are not perpendicular. However, the area of any parallelogram is given by $A = bh$, where b is the base and h is the height. The height must be perpendicular to the base. The diagram below shows why the formula works. If the shaded region within the parallelogram is moved to the left side, the figure would be a rectangle with a length of b and a width of h.

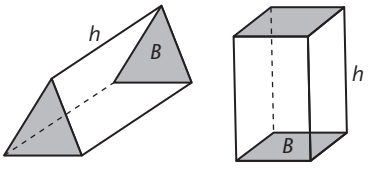
$A = bh$



Any triangle is half of a parallelogram, and the area of a triangle can be calculated using $A = \frac{1}{2}bh$. The height of the triangle must be perpendicular to the line containing the base. The height may be inside the triangle, outside the triangle, or it may be one of the legs.



Volume is the amount of space within a solid, or three-dimensional, shape. The volume tells how many unit cubes would fill the shape; therefore, volume is measured in cubic units. A **prism** is a three-dimensional shape with two congruent bases in parallel planes and at least three rectangular faces. For all prisms, the volume is the product of the area of a base and the height. The formula for the volume of a prism is $V = Bh$, where B is the area of a base, and h is the height or distance between the bases.



Example **A triangle has a base of 5 inches and covers an area of 15 square inches. What is its height?**

1) Identify the known information. The figure is a triangle with a known base and area and an unknown height. Use the formula $A = \frac{1}{2}bh$, where $A = 15$ in.^{2}, $b = 5$ in., and h is unknown.

2) Solve for h.

$A = \frac{1}{2}bh$

$15 = \frac{1}{2}(5)h$

$2(15) = (2)\left(\frac{1}{2}\right)(5)h$

$30 = 5h$

$6 = h$

The height of the triangle is 6 inches.

Example **A rectangular room has an area of 400 square feet. If the length of the room is 16 feet, what is the width of the room?**

1) Identify the known information. In this problem, the area and the length of the rectangular floor are known, but the width of the room is not known. Use the formula $A = lw$, where A is 400, l is 16, and w is unknown.

2) Solve for the unknown value. Divide the area by the given base.

$A = lw$

$400 = 16w$

$400 \div 16 = w$

$25 = w$

The width of the room is 25 feet.

Example **A concrete brick is in the shape of a rectangular prism. Its length is 8 inches, its width is 4 inches, and its volume is 64 cubic inches. What is the height of the concrete brick?**

1) Find B, the area of base. The base of a rectangular prism is a rectangle. The area of any rectangle is $l \times w$. In this shape, the area of the base is 8 inches $\times$ 4 inches or 32 square inches.

2) Substitute the known values into the formula for volume. Solve for the unknown value, h. Divide the volume by the area of the base.

$V = Bh$

$64 = 32 \times h$

$64 \div 32 = h$

$2 = h$

The height of the concrete brick is 2 inches.

Guided Practice

A doorstop is shaped like a triangular prism. The area of its triangular base is 5 square inches, and its volume is 22.5 cubic inches. What is the height of the doorstop?

A. 4.50 inches
B. 9.00 inches
C. 56.25 inches
D. 112.50 inches

Option A is correct. Substitute the known values into the volume formula $V = Bh$.

$22.5 = 5h$

$22.5 \div 5 = h$

$h = 4.5$

The height of the doorstop is 4.5 inches.

Strategy

Remember that in a triangular prism, the height of the triangular base is not the same thing as the height of the prism.

Practice

Read each question. Select the correct answer.

1 Collin is building a triangular support for a shelf. The triangle has an area of 28 square inches and a base of 7 inches. What is the height of the triangular support?

A. 4 inches
B. 8 inches
C. 14 inches
D. 21 inches

2 The area of a rectangular horse pen is 52 square meters. The width of the rectangle is 8 meters. What is the length of the pen?

A. 6.5 meters
B. 7.2 meters
C. 44.0 meters
D. 60.0 meters

3 A rectangular gameboard has an area of 6 square feet and a length of 2 feet. What is the width of the gameboard?

A. 12 feet
B. 10 feet
C. 3 feet
D. 2 feet

4 Anika has an aquarium in the shape of a rectangular prism. Its volume is 10.5 cubic feet, its length is 3.5 feet, and its width is 1.5 feet. What is the height of the aquarium?

A. 2.00 feet
B. 2.15 feet
C. 5.00 feet
D. 5.25 feet

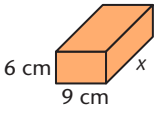
5 A ramp in the shape of a triangular prism is used on a loading dock. The volume of the prism is 1,782 cubic inches. The triangular base has a height of 9 inches and a base of 12 inches. What is the height of the ramp?

A. 8.25 inches
B. 16.75 inches
C. 33.00 inches
D. 48.00 inches

6 Albert has a shipping box in the shape of a rectangular prism. The volume of the box is 1,188 cubic centimeters. The base of the box has an area of 99 square centimeters. What is the height of the box?

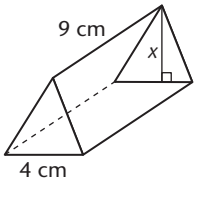
A. 6 centimeters
B. 8 centimeters
C. 10 centimeters
D. 12 centimeters

7 A brick has a volume of 1,080 cubic centimeters and dimensions as shown. What is the value of x?



A. 16 centimeters
B. 18 centimeters
C. 20 centimeters
D. 24 centimeters

8 The volume of the triangular prism is 54 cubic centimeters. What is the value of x?

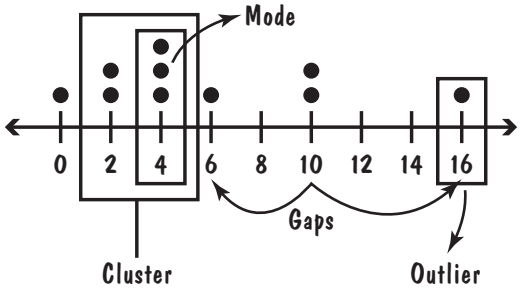


A. 3 centimeters
B. 4 centimeters
C. 5 centimeters
D. 6 centimeters

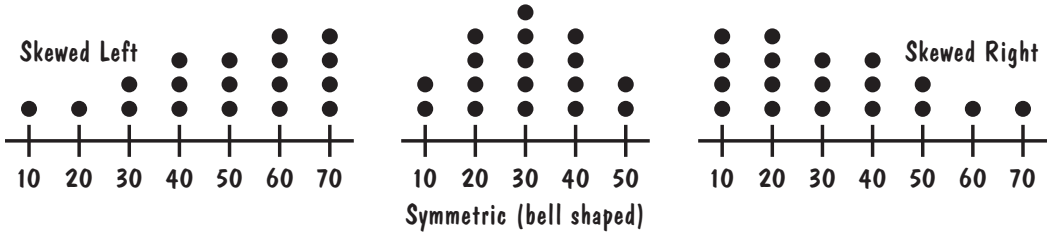
Lesson 45 Analyze Line Plots

S.ID.1

A **line plot** shows how often data points occur by plotting them on a number line. The highest point of a line plot shows the **mode**, or the value that occurs most often. If a line plot has two modes of equal height, the line plot is said to be **bimodal**. An **outlier** has a value much greater or much less than the other values in the set. A **cluster** occurs when several values are close together. A **gap** is a space with no values.



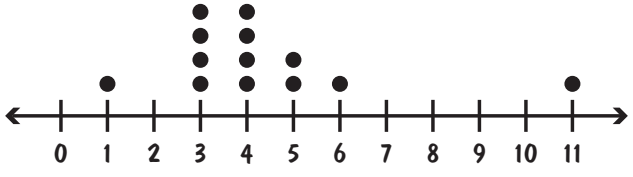
Data can be **symmetric**, **skewed left**, or **skewed right**. Look to see which side has the long tail. A long tail to the left, with more data spread out on that side of center, means the data is skewed left.



Example **Use a line plot to describe this data set. {1, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 6, 11}.**

1) Create a number line. Identify the highest and lowest values in the data set. Label a point at each end with these values: 1 and 11. Draw evenly spaced units on the number line.

2) Plot the data by making a dot (or *x*) above the matching value on the number line for each data point in the set.



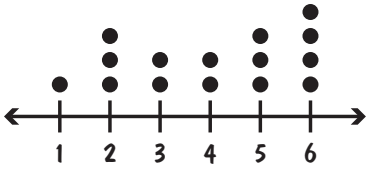
3) Identify the shape of distribution. Since there is more data to the right of the center, with a longer tail to the right, the data is skewed right.

4) Find the outliers, if there are any. The line plot makes it clear that the value of 11 is widely separated and different from the rest of the values. For this data set, 11 is an outlier.

5) Identify clusters or gaps. There is a cluster between 3 and 6. There is a gap between 6 and 11.

Guided Practice

What is the center of the data shown on this line plot?



A. 2

B. 3

C. 4

D. 5

Option C is correct. To find the median, start at the outside and move dot by dot from each end toward the center, until they meet at the median which has the same number of dots on each side.

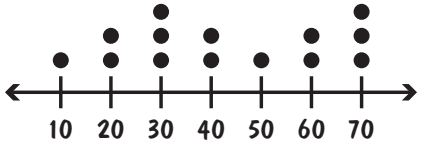
Strategy

When the shape is not symmetric, usually the median rather than the mean is used for the center. In this case, either measure gives an answer of 4.

Practice

Read each question. Select the correct answer.

1 Which data set matches the line plot?



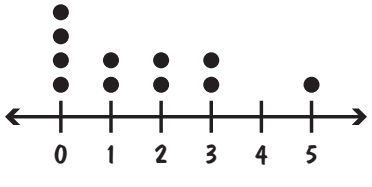
A. 10, 20, 30, 40, 50, 60, 70

B. 40, 40, 10, 20, 20, 30, 30, 30, 70, 70, 70, 50, 60, 60

C. 10, 20, 20, 30, 30, 30, 40, 40, 60, 60, 70, 70, 70

D. 20, 30, 20, 10, 30, 40, 50, 60, 70, 40, 70, 70

2 Which of the following describes the shape of this distribution?



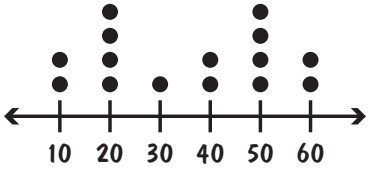
A. symmetric

B. bimodal

C. skewed left

D. skewed right

3 Which of the following describes the shape of the distribution?



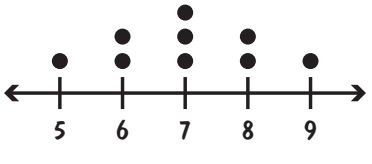
A. symmetric

B. skewed right

C. bimodal

D. skewed left

4 What is the center of distribution?



A. 5

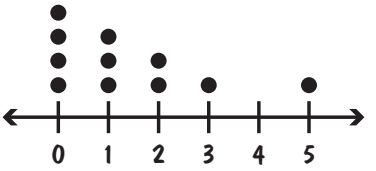
B. 6

C. 7

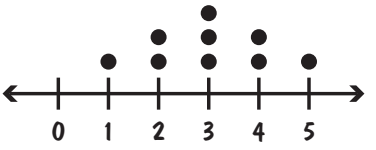
D. 8

5 Which line plot matches the data set {5, 4, 3, 1, 3, 4, 5, 5, 4, 5}?

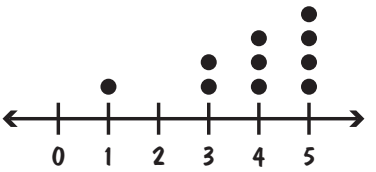
A.



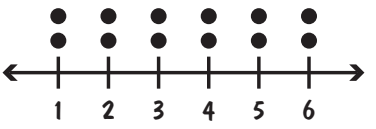
B.



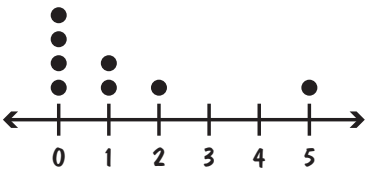
C.



D.



6 What is the outlier, if any?



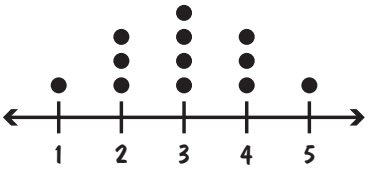
A. None

B. 0

C. 4

D. 5

7 Which description matches the line plot?



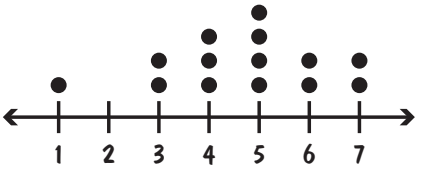
A. The data shape is skewed right. The median is 3. The data is distributed between 1 and 5, with no outliers. The range is 4.

B. The data shape is symmetrical and bell-shaped. The median is 3. The data is distributed between 1 and 5, with no outliers. The range is 4.

C. The data shape is triangular. The median is 2. The data is distributed between 1 and 5, with no outliers. The range is 5.

D. The data shape is uniform. The median is 3.2. The data is distributed between 1 and 5, with no outliers. The range is 5.

8 Data was collected by asking 14 people how many times a week they ate salad. Which conclusion is supported by the line plot ?



A. Most people eat salad less than 3 times per week, according to the survey.

B. It is unusual for people to eat salad more than 4 times per week, according to the survey.

C. Most people surveyed do not eat salad.

D. Most people surveyed eat salad at least 4 times per week.

Lesson 53 Understand Probability

7.SP.5, 7.SP.6

The probability of an event tells how likely it is that the event will happen. If an event is sure to happen (temperature in Austin, Texas reaches 100 degrees sometime in the summer), then the probability is 1. If an event is impossible (the baseball weighs more than the bowling ball), so it will never happen, then the probability is 0. Probability ranges from 0 to 1. A larger probability indicates a more likely event. A probability of 0.5 says that the event has the same chance of happening as it does of not happening.

In the language of mathematics, the sentence "The probability of drawing a red marble is 5 out of 10" is written this way: $P(\text{red}) = \frac{5}{10}$.

To find probability, write the ratio $\frac{\text{winning outcomes}}{\text{all outcomes}}$.

Example **Decide whether the event is impossible, unlikely, likely, or certain. Assign a number to that probability..**

It will snow in Phoenix, Arizona.	Snow in Phoenix is not impossible, but it is unlikely. A probability that is close to 0, but not equal to 0, represents an unlikely event. A reasonable estimate of the probability for this event is 0.1.

The table below shows some possible events for probabilities of 0, 0.5, and 1.

Probability	0	0.5	1
Tossing a coin	Getting heads and tails on one flip of a coin	Getting heads on a flip of a coin	Getting heads or tails on one flip of a coin
Rolling a six-sided number cube	Rolling an even number and an odd number on one roll of a six-sided number cube	Rolling an even number on one roll of a six-sided number cube	Rolling an even number or an odd number on one roll of a six-sided number cube
Drawing a marble from a bag	Drawing (without looking) a blue marble from a bag with 10 red marbles of identical size	Drawing (without looking) a blue marble from a bag with 5 blue marbles and 5 red marbles of identical size	Drawing (without looking) a blue marble from a bag with 10 blue marbles of identical size
Spinning a numbered spinner	Getting a number less than 1 on a spinner with 8 equal sections labeled 1–8	Getting a number greater than 4 on a spinner with eight equal sections labeled 1–8	Getting a number greater than 0 on a spinner with eight equal sections labeled 1–8.

Example **Find the probability of getting a 3 with one roll of a six-sided number cube.**

Since there are 6 numbers on a six-sided number cube, there are 6 possible outcomes. Only 1 is a winning outcome.

$P(3) = \frac{1}{6} = 0.1\overline{6} \approx 16.7\%$.

Guided Practice

Which of the following *best* represents the probability of finding a four-leaf clover?

A. 0.05

B. 0.5

C. 0.55

D. 0.75

Option A is correct. Finding a four-leaf clover is a rare occurrence, but it is not impossible to find one. So, the probability is very close to 0, but not equal to 0. Of the choices given, the value of 0.05 best represents the probability of finding a four-leaf clover.

Strategy

Use common sense to judge how likely it is that an event will happen and then assign an appropriate number to that probability.

Practice

Read each question. Select the correct answer.

1 What is the probability of rolling a number less than 10 on a six-sided number cube?

A. 0

B. 0.25

C. 0.5

D. 1

2 What is the probability of one person driving two different vehicles at the same time?

A. 0

B. 0.5

C. 0.75

D. 1

3 What is the probability of a year having 12 months?

A. 0

B. 0.12

C. 0.48

D. 1

4 Which of the following *best* represents the probability of winning the lottery?

A. 0.01

B. 0.5

C. 0.7

D. 0.99

5 Which of the following *best* represents the probability of hard-drive failure on a brand new laptop?

A. 0.03

B. 0.51

C. 0.57

D. 0.78

6 Which of the following *best* represents the probability of paying high rent in downtown New York City?

A. 0.02

B. 0.17

C. 0.52

D. 0.89

7 Which of the following *best* represents the probability of airplane engine failure?

A. 0.02

B. 0.44

C. 0.79

D. 0.92

8 Which of the following *best* represents the probability of getting heads on a coin toss?

A. 0.01

B. 0.25

C. 0.50

D. 0.99

9 Which real-world situation *best* matches a probability of 0.1?

A. passing a test after studying for 3 hours

B. finding a rare prize in a cereal box

C. temperatures reaching 110° F in Death Valley, California during the summer months

D. drawing a face card out of a standard deck of 52 cards

10 Which real-world situation *best* matches a probability of 0.9?

A. rolling 10 sixes in a row on a six-sided number cube

B. hitting the bullseye in a dart game

C. drawing an ace out of a standard deck of cards

D. drawing a number less than 10 from a stack of cards numbered 1 through 10

11 Which real-world situation has a probability of 1?

A. a tornado occurring during the summer months in Texas

B. scoring a 1,600 on the SAT exam

C. a week having 7 days

D. stocks rising 2 points over the course of a day

12 On one roll of a six-sided number cube, what is the probability of getting an even number?

A. 16.6%

B. 83%

C. 50%

D. 100%

13 What is the probability, with one spin, of getting a number less than 3 on a spinner with four equal sections labeled 1–4?

A. 50%

B. 75%

C. 25%

D. 33%

14 What is the probability, on one roll of a six-sided number cube, of getting a number greater than 6?

A. 100%

B. 75%

C. 50%

D. 0%

15 What is the probability of drawing (without looking) a blue marble from a bag with 7 blue marbles and 3 red marbles of identical size?

A. 43%

B. 100%

C. 30%

D. 70%